

them using the same radiation properties as were employed in correcting the present data. The results of all three investigations cover a nine-decade range of the Rayleigh number. To accommodate this range, three abscissa ranges, each covering three decades, are employed in the figure. The solid and long-dashed lines respectively represent the correlations of McAdams [1] and Mikheyev [4].

Examination of the figure indicates that both the present data and that of Langmuir generally fall within 10 per cent of the McAdams correlation, with a clear tendency for the data to lie below the correlating line. Whereas Petavel's data appear to scatter more than the others, it also lies below the correlation in the range of $GrPr$ that is common to all three sets of data.

In light of these observations, the McAdams correlation appears to be generally adequate for predicting low Rayleigh number natural convection from high temperature wires to gases. On the other hand, a better representation may be obtained by correlating on the basis of the present data and that of Langmuir. Such a correlation is represented by the short-dashed lines, the coordinates of which are given in Table 1.

Table 1. Coordinates of proposed correlations

$GrPr$	Nu (Fig. 1)	Nu (Fig. 2)
10^{-4}	0.463	0.439
10^{-3}	0.525	0.499
10^{-2}	0.596	0.565
10^{-1}	0.800	0.745
10^0	1.07	0.985
10^1	1.51	1.31
10^2	2.11	2.00

Except in the range of $GrPr$ between 10^{-3} and 10^{-2} , the data appear to favor the McAdams correlation over the Mikheyev correlation.

For natural convection boundary layers, the assumption of pressure invariance across the boundary layer leads to

$\beta = 1/T_\infty$ for a perfect gas. The effect of evaluating β in this way will now be examined. For this purpose, the data were reduced using the groupings defined by equation (1), with all fluid properties evaluated at the film temperature T_f but with β equal to $1/T_\infty$.

The thus-evaluated data are shown in Fig. 2, where the McAdams correlation is also indicated. The effect of the re-evaluation of β is to shift the data to the right relative to the McAdams line, thereby widening the gap in the range covered by the present data and that of Langmuir. Therefore, if it is desired to evaluate β as $1/T_\infty$, then the McAdams correlation no longer suffices. A proposed correlation of the data is indicated by the short-dashed line in Fig. 2, and the corresponding coordinates are listed in Table 1.

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TRANSIENT CONDENSATION WITHOUT CONDENSATE FLOW

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NOMENCLATURE

c ,	specific heat;
h ,	heat-transfer coefficient;
h_{fg} ,	latent heat of vaporization;
k ,	thermal conductivity;

t ,	time;
T ,	temperature;
α ,	thermal diffusivity;
δ ,	condensate (liquid) layer thickness;
λ ,	defined by equation (3) or (5);
ρ ,	liquid density.

Subscripts

l ,	liquid;
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s, solid;
 sat, saturated vapor;
 w, wall.

ROHSENOW in the recent handbook edited by Rohsenow and Hartnett [1] considers two examples of transient condensation in which there is no flow of condensate; namely, the bottom of a container and zero gravity. Gebhart also considered this same problem [2]. Rohsenow and Gebhart solve the example of condensation on the bottom of a container, with the surface maintained at $T_w < T_{\text{sat}}$ by neglecting the heat capacity of the liquid layer. They, therefore, assume that the temperature distribution is linear in the liquid layer. Their approximate solution for the transient liquid layer thickness,

$$\delta = \left(\frac{2k(T_{\text{sat}} - T_w)t}{\rho h_{fg}} \right)^{1/2}, \quad (1)$$

is identical to the solution attributed to Stefan for the solid layer thickness in the freezing problem {equation (21) of [3]}.

It is the purpose of this note to point out that an exact solution exists to this problem. The solution is Neumann's classical solution to the Stefan or freezing problem [3], specialized to the case in which the freezing liquid is initially at its melting temperature. The solution is

$$\delta = 2\lambda(\alpha_l t)^{1/2}, \quad (2)$$

where λ is the root of the transcendental equation

$$\lambda e^{\lambda^2} \operatorname{erf} \lambda = \frac{c_l(T_{\text{sat}} - T_w)}{h_{fg}\pi^{1/2}}. \quad (3)$$

The previous approximate solution is the asymptotic solution for $h_{fg}/(c_l(T_{\text{sat}} - T_w)) \rightarrow \infty$.

It can be shown that the heat-transfer coefficient for this problem is

$$h = \frac{k_l}{\operatorname{erf} \lambda (\pi \alpha_l t)^{1/2}}. \quad (4)$$

Thus, at $t = 0$ the heat-transfer coefficient is singular and strongly time dependent.

Other exact solutions which can be applied to condensation problems are available in Carslaw and Jaeger. One solution is for transient condensation on a semi-infinite container-bottom initially at temperature $T_w < T_{\text{sat}}$. The solution considers transient conduction in the container-bottom. For this problem, equation (2) again gives the liquid layer thickness; the transcendental equation for λ becomes

$$\lambda e^{\lambda^2} \left\{ \frac{k_l \alpha_s^{1/2}}{k_s \alpha_l^{1/2}} + \operatorname{erf} \lambda \right\} = \frac{c_l(T_{\text{sat}} - T_w)}{h_{fg}\pi^{1/2}}. \quad (5)$$

It is thus apparent that many solutions to change of phase problems which have been used to model solid-liquid melting-freezing problems can be used to model liquid-vapor condensation-evaporation problems provided there is no motion in the liquid phase. Sources of solutions include Carslaw and Jaeger, and Goodman's review article on approximate integral solutions [4].

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